

Robust PID Controller for Quad-rotors

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Abstract— This study considers robust proportional integral derivative (PID) control for attitude stabilization of quad-rotors. The proposed method is designed for maintaining robustness against uncertainty of the system parameters and minimizing the control input for the quad-rotors. The closed-loop poles of the controlled system are placed within a region with specified decay rate and damping ratio for achieving specified transient response. The PID gains are tuned offline using H_∞ theory. The robust control design problem is converted to optimization problem and solved using genetic algorithm. Finally, the proposed controller is simulated in the nominal and perturbations scenarios.

Keywords— attitude control, PID tuning, quad-rotors, robust control

I. INTRODUCTION

QUAD-ROTORS are aircrafts lifted and propelled using four rotors. It has no fixed wings, no elevators, and no ailerons. Quad-rotors are totally lifted and moved using the thrust created by the four rotors/propellers. The configuration of a quad-rotor is shown in **Figure 1**. The direction of propellers 1 and 3 are opposed with the propellers 2 and 4. Therefore at the same speed, different torques directions compensate each other.

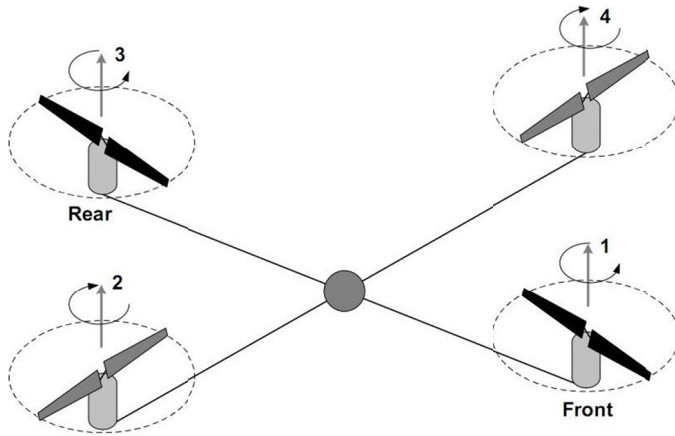


Figure 1 – Configuration of a quad-rotor

There are four basic movements of quad-rotors: translating on z-axis, and turning around its x-axis (roll), y-axis (pitch), and z-axis (yaw). In order to raise or lower the height position of the quad-rotor, the quad-rotors have to increase or decrease all of the propellers with the same amount of speed. For turning along its z-axis (yaw), the quad-rotors have to decrease the velocities of two opposed propellers, and increase the others two propellers. Therefore, the quad-rotors have the same lift force but it turn to the direction that has a bigger torque. For rotations along its x-axis (roll), the quadrotors have to increase the velocity of the propellers 2 or 4, and decrease the velocities of the opposed propellers. For creating the pitch movement, the quad-rotors increase the velocities of the propellers 1 or 3, and decrease the velocities of the opposed propellers.

In this paper, we consider to control micro quad-rotors. The references in this topic can be found in many papers. Ref. [1]-[4] considered sliding mode control for quad-rotors. Ref. [5]-[9] used back-stepping method for control the quad-rotor. The feedback linearization method is used to control quad-rotors in [10]-[12]. The Adaptive control is used for controlling the quad-rotors in [13]. Many papers have been published relating to PID control for quad-rotors. Among these, [14]-[20] gave unclear procedure how they tuned the PID gains. Other papers, for example [21], used conventional Ziegler-Nichols method, which is known as fragile controller [22], for tuning the PID gains for the quad-rotor. Ref. [23] and [24] used PID for the nonlinear model of quad-rotor. Fuzzy gain-scheduled PID is used in [25].

In particular, we consider robust PID controller design for attitude stabilization of quad-rotors using H_∞ theory. The proposed method has advantage compare to conventional robust H_∞ control where the full order controller is constructed from its application. We choose the PID controller because the controller is widely used as a default controller in micro UAVs. Therefore, we keep the original hardware/software structure in the flight controller of UAVs. We use robust control strategy rather than online control methods due to low computation cost for implementing the robust PID control. The control objectives of the proposed method are providing robustness against parameter uncertainties of the model and minimizing the control input for the quad-rotors. The parameters uncertainties can be happened due to mismatch model and/or additional payload for the quad-rotors. The closed-loop poles of the controlled systems are shifted to a region with a specified decay rate and damping ratio for maintaining transient performance of the controlled system.

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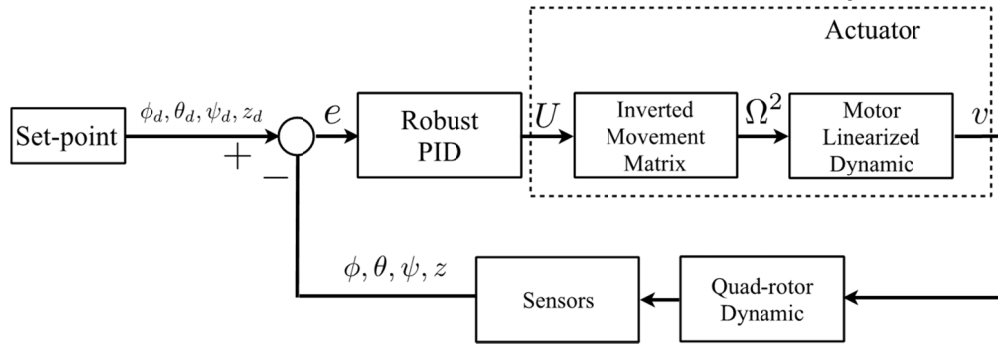


Figure 2 – Feedback control system for quad-rotors

This paper is organized as follows. Dynamic model of the controlled system is explained in Section II. The proposed robust PID controller is given in Section III. The computer simulation results using the proposed controller are presented in Section IV. Finally, conclusion and future work are given in Section V.

II. DYNAMIC MODEL OF QUAD-ROTORS

The block diagram of the proposed robust PID controller for quad-rotors is shown in **Figure 2**. The control input U is calculated using the error e between the set-point references and measured outputs. The actuator (i.e., rotor) converts U to voltage v via the inverted movement matrix and the motor's linearized dynamics. The inverted movement matrix changes U to squared speed of the propeller Ω . The motor's linearized dynamic changes Ω to voltage in the rotor. In this section, the quad-rotor's dynamic, the inverted movement matrix, and the motor's linearized dynamic are discussed. **Table 1** summarizes the variables that are used in this paper.

Table 1 – List of variables

Symbol	Unit	Description
x, y, z	m	Position in x, y, z wrt e-frame
ϕ, θ, ψ	rad	Roll, pitch, yaw, wrt e-frame
ϕ_d, θ_d, ψ_d	rad	Desired roll, pitch, yaw
z_d	m	Desired height
U_i	Nm	Throttle, roll, pitch, yaw force
m	kg	Mass of quad-rotor
p, q, r	rad s^{-1}	Angular vel. in x, y, z wrt b-frame
g	m s^{-2}	Gravity acceleration constants
I_{xx}, I_{yy}, I_{zz}	N m s^2	Moi in x, y, z axis
Ω	rad s^{-1}	Overall propeller speed
Ω_i	rad s^{-1}	Propeller speed in rotor i
v_i	V	Voltage in rotor i
J_{TP}	N m s^2	Total rotat. moi around prop. axis
b	N s^2	Thrust factor
l	m	Dist. from center of quad. to rotor
d	N m s^2	Drag factor
J_P	N m s^2	Rotat. moi around prop. axis
η	-	Gear box efficiency
N	-	Gear box reduction ratio
J_M	N m s^2	Rotation moi around motor axis
K_M	N m A^{-1}	Mechanic motor constant
K_E	V s rad^{-1}	Electric motor constant
R	Ohm	Motor resistance

Note: e-frame (earth-frame); b-frame (body-frame)
wrt (with respect to); moi (moment of inertia); $i = 1, 2, 3, 4$

A. Dynamic model of quad-rotor.

Equation (1) shows the quad-rotor's acceleration according to the basic movement commands [20].

$$\begin{aligned}\ddot{x} &= (\sin \psi \cos \phi + \cos \psi \sin \theta \cos \phi) U_1 / m \\ \ddot{y} &= (-\sin \psi \cos \phi + \cos \psi \sin \theta \cos \phi) U_1 / m \\ \ddot{z} &= -g + (\cos \theta \cos \phi) U_1 / m\end{aligned}\quad (1)$$

$$\begin{aligned}\dot{p} &= ((I_{yy} - I_{zz}) / I_{xx}) q r - (J_{TP} / I_{xx}) q \Omega + U_2 / I_{xx} \\ \dot{q} &= ((I_{yy} - I_{zz}) / I_{yy}) p r + (J_{TP} / I_{yy}) p \Omega + (U_3 / I_{yy}) \\ \dot{r} &= ((I_{xx} - I_{yy}) / I_{zz}) p q + U_4 / I_{zz}\end{aligned}$$

The latter equation can be simplified using the following considerations:

- In a condition close to the hovering motion, only small angular changes occur (especially in the roll and pitch). Therefore, the angular acceleration equations can be simplified due to small value of some terms in the equations.
- The angular accelerations are referred to the angles of the quad-rotor measured in its fixed frame. In hovering motion, these angles are equivalent to the Euler angles.
- In this study, we only consider to stabilize attitude (Euler angles) and height. Therefore, the equations for describing x and y positions are omitted.

Using the considerations, (1) can be simplified as the following equation.

$$\begin{aligned}\ddot{z} &= -g + (\cos \theta \cos \phi) U_1 / m \\ \ddot{\phi} &= U_2 / I_{xx}\end{aligned}\quad (2)$$

$$\begin{aligned}\ddot{\theta} &= U_3 / I_{yy} \\ \ddot{\psi} &= U_4 / I_{zz}\end{aligned}$$

B. Inverted movement matrix

The block converges the control force (U_1, U_2, U_3, U_4) to the squared speed of the propellers $(\Omega_1, \Omega_2, \Omega_3, \Omega_4)$. The conversion is described by the following equations.

$$\begin{aligned}\Omega_1^2 &= \frac{1}{4b} U_1 - \frac{1}{2bl} U_3 - \frac{1}{4d} U_4 \\ \Omega_2^2 &= \frac{1}{4b} U_1 - \frac{1}{2bl} U_2 - \frac{1}{4d} U_4\end{aligned}$$

$$\Omega_3^2 = \frac{1}{4b}U_1 - \frac{1}{2bl}U_3 - \frac{1}{4d}U_4$$

$$\Omega_4^2 = \frac{1}{4b}U_1 - \frac{1}{2bl}U_2 - \frac{1}{4d}U_4$$

C. Linearized dynamic of the motor

This section relates the squared speed of the propellers to its voltage. There are several approaches to obtain the relationship. The first approach is conducted by solving the following equation numerically.

$$(J_p + \eta N^2 J_M) \dot{\Omega} = -\frac{K_E K_M}{R} \eta N^2 \Omega - d\Omega^2 + \frac{K_M}{R} \eta N v$$

The second method is obtained by linearizing the latter equation for simplifying the computation. In the third method, direct linear relationship between Ω and v is obtained by conducting experiment for the motor's behavior. In this study, we use the latter method for obtaining the relation between Ω and v .

III. PROPOSED CONTROL DESIGN

The proposed PID controller for quad-rotors is depicted in **Figure 3(a)**. Here, P represents the nominal plant of the quad-rotor. The inverse additive perturbation is used for representing the parameter uncertainties of the quad-rotor's model. Variable Δ represents unstructured uncertainties of the model. Controller C is the robust PID controller in the form of

$$C = k_p + \frac{k_i}{s} + k_d \frac{Ns}{s + N}$$

Variables k_p , k_i and k_d are the PID gains and N is positive number. The first order filter in the derivative term of C is used for avoiding the high level of controller output due to sudden change in the controller input. Variables sp , out , od , and u represent the reference input, plant output, output of the uncertainty, and control force, respectively. Remark that others structure of PID controller can be used and its parameters are tuned using the proposed method.

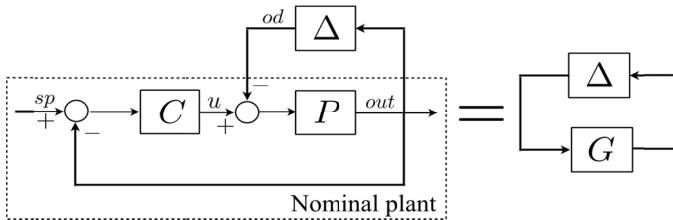


Figure 3 – The proposed control system configuration

Robust H_∞ control theory is used to tune the robust PID gains for the quad-rotor [26][27]. The controlled system configuration in **Figure 3(a)** can be simplified to the **Figure 3(b)**, where variable G is represent plant with the controller and can be expressed as

$$G = P(I - PC)^{-1}$$

Using the Small-Gain Theorem, we have the following theorem:

Theorem 1. [28],[29] For stable $\Delta(s)$, the closed-loop system is robustly stable if $C(s)$ stabilizes the nominal plant and the following holds

$$\|\Delta(I - PC)^{-1}\|_\infty < 1$$

or, in the strengthened form,

$$\|P(I - PC)^{-1}\|_\infty < \frac{1}{\|\Delta\|_\infty}$$

In order to obtain a PID controller that stabilizes the largest possible set of perturbations in Δ , it is clear that we need to solve the following minimization problem

$$\min \|P(I - PC)^{-1}\|_\infty \quad (3)$$

In the real application, DC power source for implementing U has its maximum limit in voltage. We need to put this constraint into our design strategy. For the nominal system (without presence of Δ) in **Figure 3(a)**, the transfer function from u to sp is expressed by

$$u = \frac{C}{(I + PC)} sp$$

Minimizing the ∞ -norm of the latter equation for the less energy control, we formulate this as

$$\min \|C(I + PC)^{-1}\|_\infty \quad (4)$$

In order to improve the transient performance of the controlled system, the poles of the closed-loop system have to be placed within certain region in s -plane. In this study, the closed-loop poles are placed within the D-shape region characterized by $\zeta_{spec} \leq \zeta$ and $\sigma \leq \sigma_{spec}$ as shown in **Figure 4**. Here, ζ , σ , ζ_{spec} , and σ_{spec} are the damping ratio, real part position of the closed-loop poles, maximum damping ratio, and maximum value of real part of the closed-loop system, respectively.

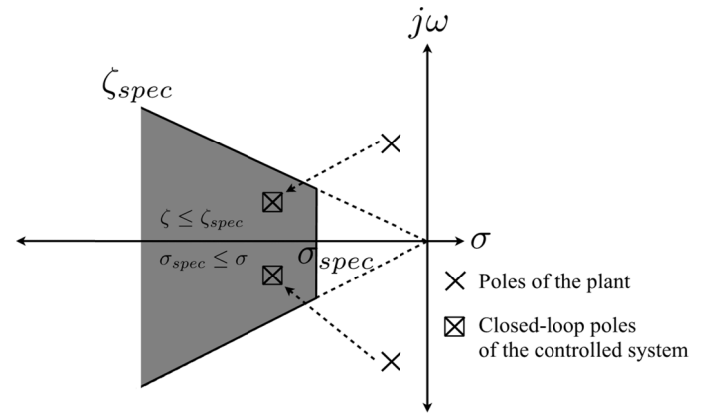


Figure 4 – D-shape region

Therefore, the design of PID controller for the quad-rotor can be formulated as the following optimization problem.

$$\text{minimize } \left\| \frac{P(I - PC)^{-1}}{C(I + PC)^{-1}} \right\|_\infty \quad (5)$$

$$\text{subject to } \zeta \leq \zeta_{spec}, \quad \sigma \leq \sigma_{spec} \quad (6)$$

Genetic Algorithm (GA) is used for solving the optimization problem in (5). In particular, the Genetic Algorithm Optimization Toolbox (GAOT) is utilized as the optimization tool [30]. The GAOT is selected because it can handle flexible form of evaluation functions and constraints. This enables us for optimizing the mixing sensitivity in (5) and for using any structure of controllers. This kind of flexibility cannot be found if we use the standard Matlab Robust H_∞ Toolbox, which can only handle the optimization of mixing sensitivity for full order controller. Others optimization tools, such as particle swarm optimization (PSO), can be used also for solving the optimization problem in (5). The PSO developed in [31] offers the same flexibility as the GAOT. Therefore, the PSO can be selected as alternative solver for the optimization.

The off-line tuning of PID gains using is conducted by using the following steps:

Step 1: Generate the objective function for GA optimization, based on (5) and (6).

Step 2: Initialize the search parameter for GA. Define genetic parameters such as population size, crossover, mutation rate, and maximum generation.

Step 3: Randomly generate the initial solution.

Step 4: Evaluate objective function of each individual in (5) and (6).

Step 5: Select the best individual in the current generation. Check the maximum generation.

Step 6: Increase the generation.

Step 7: While the current generation is large than the maximum generation, create new population using the genetic operators and go to **Step 4**. If the current generation is the maximum generation, then stop.



Figure 5 – Astec pelican quadrotor

IV. SYSTEM IDENTIFICATION AND SIMULATION RESULT

For demonstrating the effectiveness of the proposed method, computer simulations are conducted based on actual parameters of a quad-rotor. In this study, we use a Pelican Quad-rotor from Astec. The physical system of the Astec pelican is depicted in Figure 5.

A. Parameters identification and simulation setup

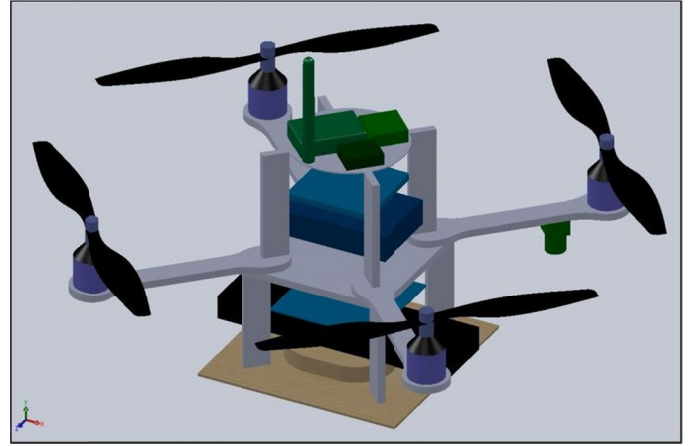


Figure 6 – Solidwork drawing for the Astec pelican quadrotor

As described in Section II, the model of quad-rotors is parameterized by the constants of moment of inertia. Solidworks software is used for identifying these constants. The Solidwork requires the precise drawing of the quad-rotor's components and its materials properties. We disassembled the quad-rotor for measuring the dimension and weight of its components. We neglect cables other small components because the total mass of these components is small compare with the total mass of quad-rotor. The integrated drawing of the quad-rotor using Solidworks is shown in Figure 6. Using the software, we can obtain the following inertia matrix:

$$I = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix} = \begin{bmatrix} 0.01186 & 0.000023 & 0.000084 \\ 0.000023 & 0.01086 & 0.000011 \\ 0.000084 & 0.000011 & 0.01652 \end{bmatrix} \text{ kg.m}^2$$

We can see that I_{xy} , I_{xz} , and I_{yz} are very small compare to I_{xx} , I_{yy} , and I_{zz} , thus we can neglect its values. Therefore, the final form of the inertia matrix is

$$I = \begin{bmatrix} 0.01186 & 0 & 0 \\ 0 & 0.01086 & 0 \\ 0 & 0 & 0.01652 \end{bmatrix} \text{ kg.m}^2$$

The linear relation between the squared velocity of rotor and its voltage can be found in its manufacture's data sheet. The linear relation is described by the following equation.

$$v_i = \frac{\Omega_i}{96.76}$$

In this study, we stabilize the attitude angles in hovering motion. Therefore, we design the robust PID control for each angle in (2). We do not design the PID control for controlling altitude of quad-rotor. In (2), the value of U_1 is given slightly bigger than the weight of the quad-rotor. Figure 7 shows the

testbed for conducting such experiment for the quad-rotor. Using this testbed, the altitude of the quad-rotor is held by the testbed. The control of the attitude stabilization is conducted simultaneously because the calculation of voltage in each rotor is combination from U_1 , U_2 , U_3 , and U_4 . The DC power source for the quad-rotor is given by four cells battery. Therefore, the maximum voltage for this battery is 14.8 V.



Figure 7 – Testbed for testing attitude stabilization of the quad-rotor

B. Simulation result

Table 2 – The obtained PID gains

	k_p	k_i	k_d
Roll	1.0142	1.1987	0.3029
Pitch	1.0092	1.1998	0.3011
Yaw	1.0015	1.1969	0.3007

Table 3 – The position of the closed-loop poles

Nominal Case:	Closed-loop poles position
Roll angle	$-54.1; -42.3; -1.8 \pm 1.1j$
Pitch angle	$-48.2 \pm 13.9j; -1.8 \pm 1.1j$
Yaw angle	$-76.4; -19.8; -1.9 \pm 1.1j$
+50% Case:	Closed-loop poles position
Roll angle	$-78.5; -17.7; -1.9 \pm 1.1j$
Pitch angle	$-75.9; -20.3; -1.9 \pm 1.1j$
Yaw angle	$-85.9; -9.9; -2.1 \pm 1.1j$

For the performance design specification, we set the maximum value of the real part and the damping ratio as

$$\sigma_{spec} = -1 \quad \zeta_{spec} = 0.2$$

The GAOT is used for solving the optimization in (5). The searching interval of the PID gains is set from 0.01 to 5. The GAOT creates 500 generations for searching the admissible PID gains. Using these configurations, approximately 22 seconds are needed by the GAOT for solving the optimization problems. The obtained PID gains for stabilization the roll, pitch, yaw angles are given in Table 2. The simulations are conducted using the nominal values of the moment of inertia and its perturbations up to +50%. The initial angles for simulating the angles stabilization are 10 degree for roll/pitch angles and 5 degree for yaw angle.

The simulation result on attitude stabilization using the obtained PID gains is shown in Figure 8. The nominal system

and its perturbation reach the steady state within one second. The good transient performances are achieved because the closed-loop poles systems are successfully placed within specified D-shape region. The location of the closed-loop poles for the nominal system and its perturbations are presented in Table 3. The fast transient responses are reached without using the excessive control input in each rotor. The maximum voltages in all rotors are still below the maximum voltage as shown in Figure 9. Hence, the simulation results show the effectiveness of the proposed method for providing robust PID control for the angles stabilization of the quad-rotor.

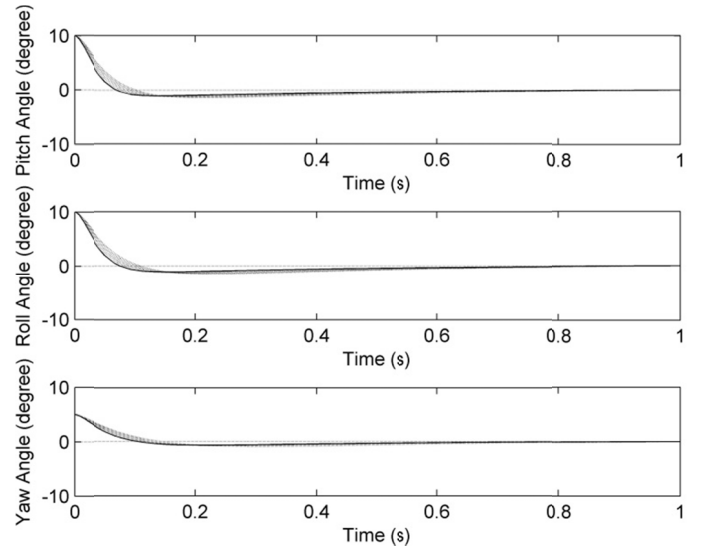


Figure 8 – Attitude angles of the nominal system (solid) and its perturbations (dot)

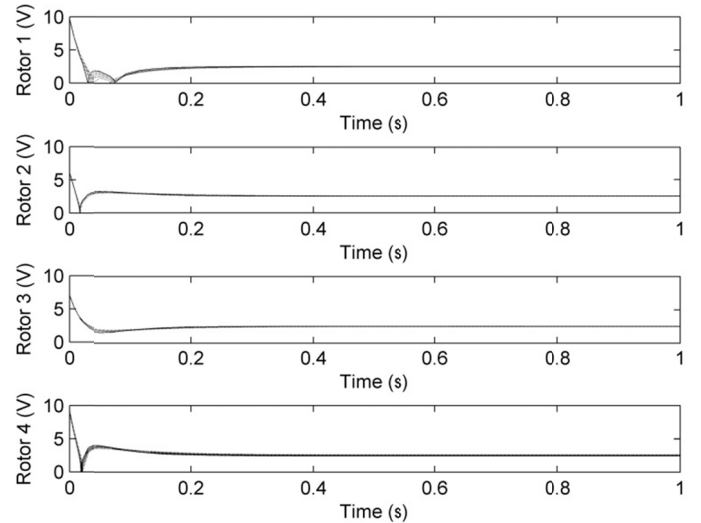


Figure 9 – Voltage in each rotor for the nominal system (solid) and its perturbations (dot)

V. CONCLUSION AND FUTURE WORK

The proposed robust PID design for attitude stabilization of the quad-rotor is discussed. The PID gains are tuned using the H_∞ theory. The mixing sensitivity problem in robust H_∞ norm is converted to the optimization problem and solved using GA algorithm. The obtained PID gains successfully maintain the system performance against the parameter perturbations. The

closed-loop poles of nominal system and its perturbation are placed within the specified D-region. The obtained controller also successfully kept the level of the control input below the maximum allowable voltage in its rotors in nominal and perturbed cases. Hence, the proposed control design provides an effective and practical tool for robust PID tuning for attitude stabilization of quad-rotors.

In the future, the experiment will be conducted using the proposed control design. Intuitively, the implementation of this control technique will follow the simulation result because the structure of controller is simple and fits with default controller. Additionally, the proposed control technique gives robustness properties to the controlled system. Therefore, the mismatch between model and physical system will not affect in real implementation. The control of translational motion along x , y , and z axis using the proposed technique will also be considered in the future.

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